1.7 Work Done, Potential and Kinetic Energy

Students should be able to:

1.7.1 Define work done, potential energy, kinetic energy, efficiency and power

1.7.2 Recognise that when work is done energy is transferred from one form to another

1.7.3 Calculate the work done for constant forces, including forces not along the line of motion

1.7.4 Recall and use the equations $\Delta p.e. = mg\Delta h$ and $k.e. = \frac{1}{2} mv^2$

1.7.5 State the principle of conversation of energy and use it to calculate the exchanges between gravitational potential energy and kinetic energy

1.7.6 Recall and use: P = work done ÷ time taken. P = Fv and

$$\text{Efficiency} = \frac{\text{useful energy (power) output}}{\text{energy (power) input}}$$

Work

When energy is transferred from one form to another it may be transferred by doing work. For example, when you lift an object you do work by transferring chemical energy to kinetic energy and gravitational potential energy. This concept of work gives us a way of defining energy.

Energy is defined as the stored ability to do work.

When we say that, for example, a battery stores 50 000 joules of energy, we simply mean that the battery has the capacity to do 50 000 joules of work. But what do we mean by 'work'?

We define the work done by a constant force as the product of the force and the distance moved in the direction of the force.

Work done = constant force × distance moved in the direction of the force

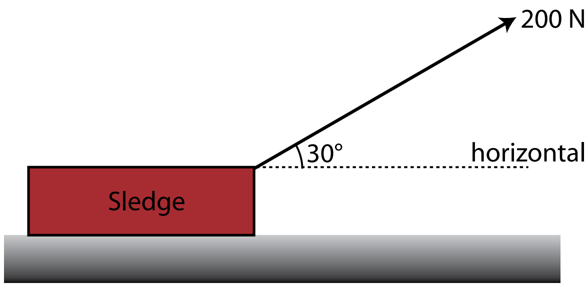
or $W = F \times s$

where W = work done in joules (or Nm)
F = constant force in N
s = distance moved in the direction of the force in m

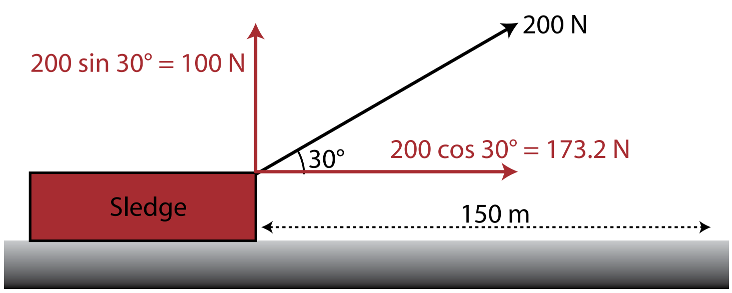
At GCSE you did not pay much attention to the words 'in the direction of the force' used in the definition of work. At AS level it is important that you recognise and can apply this new definition when the force and the distance moved are not in the same direction. The following worked example shows why it is important to take into account the direction of the motion.

Worked Example

Consider an Arctic explorer dragging a sledge across a frozen lake. The explorer attaches the rope to his waist and the force of 200 N is applied at 30° to the horizontal. We can model this situation in the diagram below. How much work is done by the explorer in dragging the sledge 150 metres across the ice at a steady speed?



The difficulty here is that the force, F (200 N), and the displacement, s, are not in the same direction. The easiest solution is to resolve the 200 N force into its vertical and horizontal components as shown below.

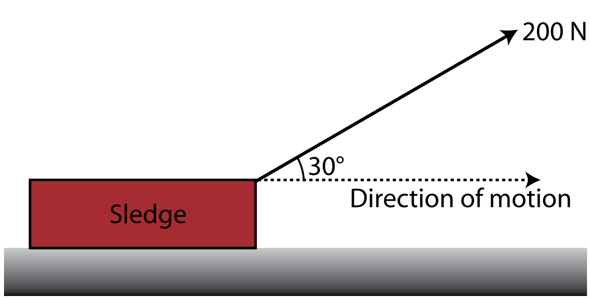


Provided the sledge does not rise above the ice, no work is done by the vertical force of 100 N. **Work is done only by the horizontal component of the applied force** (173.2 N).

Work done = constant force × distance moved in direction of the force = 173.2 N × 150 m = **25 980 J**

Notice that since the sledge is moving at a steady speed there is no resultant force, so there is a frictional force of 173.2 N acting to the left. This is why we say 'the explorer is doing work against the frictional force'.

The situation described above occurs quite often and it is sometimes easier to use the general formula applicable when the force and distance moved are not in the same direction.



The general formula is:

$$W = Fs \cos \theta$$

where θ is the angle between the force and the direction of the motion.

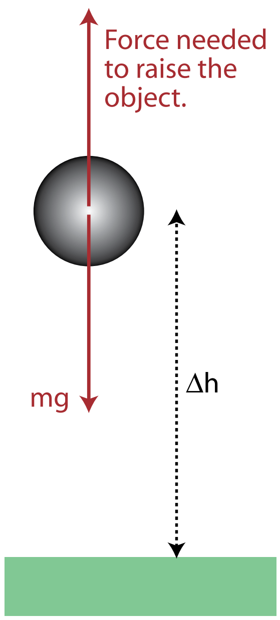
To apply this formula to the sledge example above we would write:

$W = Fs \cos \theta = 200 \times 150 \cos 30° = 200 \times 129.9 = \mathbf{25\ 980\ J}$

Potential energy

An object has gravitational potential energy when it is raised above the ground.

The gain in gravitational potential energy is equal to the work done in raising the object. If the object has zero potential energy when it is on the ground then the work done equals the amount of potential energy the object has when it is a height Δh above the ground.



If the object of mass m is raised a distance Δh then the amount of work done in raising the object is:

Work done = force needed × distance moved in direction of this force
= weight × vertical distance moved
= $mg\Delta h$ (where g is the acceleration of free fall)
= gain in gravitational potential energy

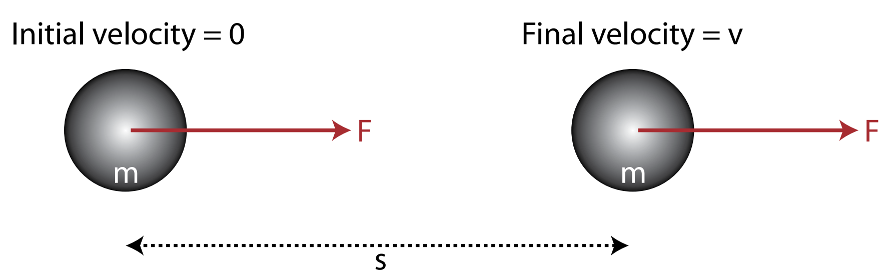
$\Delta p.e. = mg\Delta h$

where Δp.e. = change in potential energy in J
m = mass in kg
g = acceleration of free fall
Δh = vertical distance moved in m

Kinetic energy

A moving object possesses kinetic energy.

An object of mass m is initially at rest. It is acted upon a resultant force F and the object accelerates. This force acts over a distance s. Having travelled this distance the object has a velocity v.



Work done in moving the object a distance s is $W = F \times s$

Newton's 2nd law ($F = ma$) allows us to replace F in this expression, $W = ma \times s$

The equation of motion $v^2 = u^2 + 2as$ allows us introduce velocity in our expression for W. The initial velocity of this object was 0 so **$as = \frac{1}{2} v^2$**

k.e. = $\frac{1}{2} mv^2$

where k.e. = kinetic energy in J
m = mass in kg
v = velocity in ms^{-1}

In general the work done on an object is equal to the **change** in the kinetic energy of the object. If the final speed v is greater than the initial speed u then W is the work done in accelerating the object over a distance s. If the final speed v is less than the initial speed u then W is the work done in slowing the object down over a distance s.

$$W = \tfrac{1}{2} mv^2 - \tfrac{1}{2} mu^2$$

where W = work done in J
m = mass in kg
v = final velocity in ms^{-1}
u = initial velocity in ms^{-1}

Principle of Conservation of Energy

The Principle of Conservation of Energy states that energy cannot be created or destroyed but can be changed from one form to another.

Some forms of energy are more useful than others; they are more suitable for doing work and changing into other forms of energy. Electrical and chemical energy are in this category and are sometimes known as high-grade forms of energy.

On the other hand internal energy, i.e. the kinetic energy of gas molecules due to their random motion is a low-grade form of energy that is not easily converted to other forms.

The Principle of Conservation of Energy as it applies to a falling object

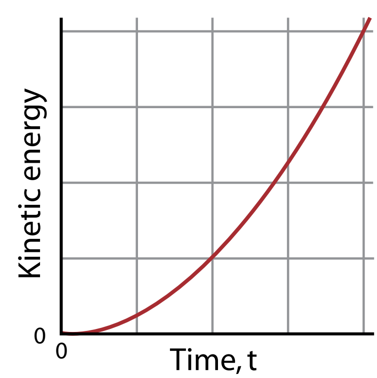
An object held above the ground and then released will gradually convert potential energy to kinetic energy. At any time its total energy (E_T) i.e. the sum of its kinetic (E_k) and potential (E_p) energies, is constant. At any point along its path, as it falls, the total energy E_T is also constant.

How does the kinetic energy and the potential energy vary with the vertical distance it falls?

As the object accelerates from rest (u=0) its velocity at any instant is v = at.

$E_k = \tfrac{1}{2} mv^2 = \tfrac{1}{2} ma^2t^2$

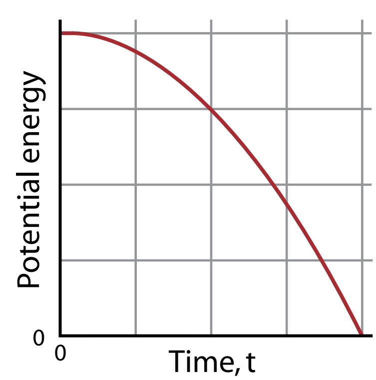
E_k is proportional to t^2.



As the object falls its E_p decreases. At any instant the E_p equals the initial potential energy (E_T) less the kinetic energy (E_k)

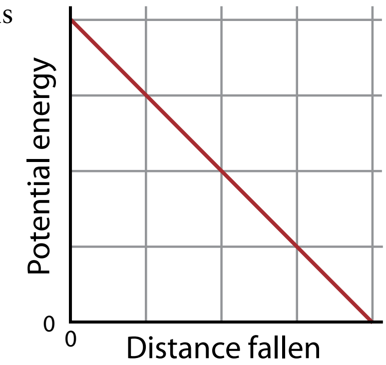
$E_p = E_T - E_k$

$E_p = E_T - \tfrac{1}{2} ma^2t^2$



If the object falls a distance y then its potential energy at this point is

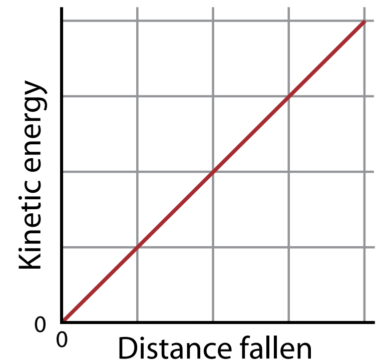
$$E_p = E_T - mgy$$



As the object falls a distance y we can use the equation of motion $v^2 = u^2 + 2as$ to find its velocity at this point. This gives $v^2 = 2gy$

$E_k = \frac{1}{2} mv^2$, but we can substitute, giving

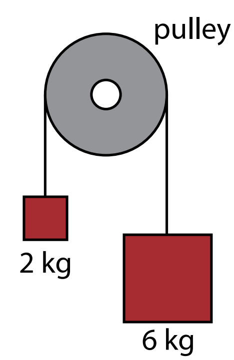
$E_k = mgy$



The following shows the strength of using energy interchange to solve problems in mechanics.

Worked Examples

1 Masses of 6.0 kg and 2.0 kg are connected by a light inextensible string passing over a smooth pulley. The string is taut when the masses are released. The smaller mass accelerates upwards and the bigger mass accelerates downwards. Using the Principle of Conservation of Energy, calculate the speed of the masses when the larger one has descended 2.0 m.



Solution

The 2.0 kg mass is accelerating upwards and is gaining both kinetic and potential energy. The 6.0 kg mass is accelerating downwards and is gaining kinetic energy and losing potential energy.

Let the speed of each mass be v (in ms^{-1}) when the larger one has descended 2.0 m.

Net loss in energy of 6.0 kg mass = p.e. – k.e. = $mgh - \frac{1}{2} mv^2 = 6 \times g \times 2 - \frac{1}{2} \times 6 \times v^2$
$= 12g - 3v^2$

Net gain in energy of 2.0 kg mass = p.e. + k.e. = $mgh + \frac{1}{2} mv^2 = 2 \times g \times 2 + \frac{1}{2} \times 2 \times v^2$
$= 4g + v^2$

By the Principle of Conservation of Energy:
the net loss in energy of the 6.0 kg mass = net gain in energy of the 2.0 kg mass

$12g - 3v^2 = 4g + v^2$, which rearranges to give

$8g = 4v^2$, which simplifies to

$v = \sqrt{(2g)} = \sqrt{(2 \times 9.81)} = \mathbf{4.4\ ms^{-1}}$

An equally valid approach is to use the idea that the total loss in p.e. is equal to the total gain in k.e.

This leads to:

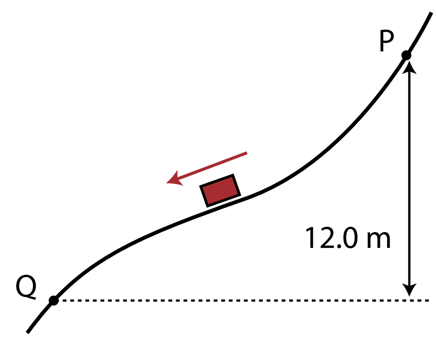
Loss in p.e. = $(mgh)_{\text{for 6 kg mass}} - (mgh)_{\text{for 2 kg mass}} = 6 \times g \times 2 - 2 \times g \times 2 = 8g$

Gain in k.e. = $(\frac{1}{2}mv^2)_{\text{for 6 kg mass}} + (\frac{1}{2}mv^2)_{\text{for 2 kg mass}} = \frac{1}{2} \times 6 \times v^2 + \frac{1}{2} \times 2 \times v^2 = 4v^2$

Hence, $8g = 4v^2$, which simplifies as above to give v = **4.4 ms^{-1}**

Note: This problem can also be solved by first finding the common acceleration of the masses and then applying Newton's equations of uniform acceleration.

2 A small block of wood passes through point P at a speed of 2.00 ms^{-1} and slides down a smooth curved track.



(a) Calculate the speed of the block as it passes point Q, 12.0 m vertically below P.

(b) Explain why it would be inappropriate to use Newton's equations of uniform acceleration in this situation.

(c) Does the time taken to travel from P to Q depend on the equation of the curved slope? Explain your answer.

Solution

(a) Let the speed of the block as it passes Q be v.
By Principle of Conservation of Energy,
Loss in gravitational p.e. = gain in k.e.

$mg\Delta h = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Substituting,	$m \times 9.81 \times 12$	$= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
cancelling m,	117.72	$= \frac{1}{2}(v^2 - 2^2)$
and solving.	v	$= 15.5\ ms^{-1}$

(b) The acceleration is not uniform (and not in a straight line).

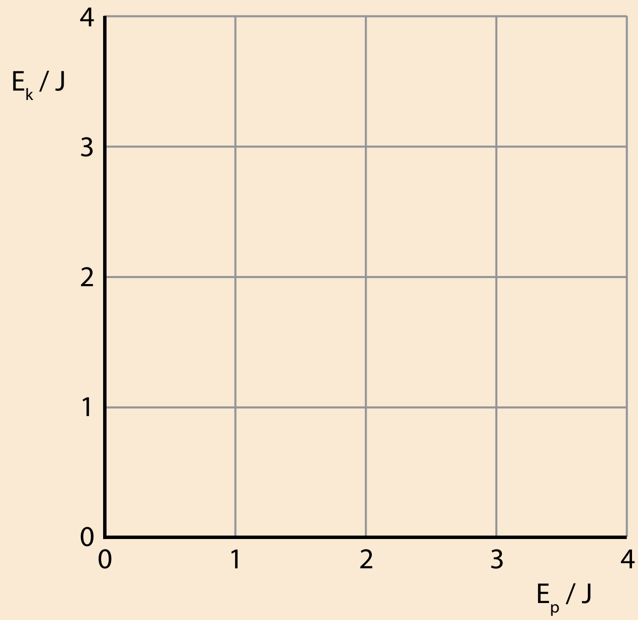
(c) Yes. The steeper the slope the greater the average acceleration and the smaller the time taken.

Exercise 14

Examination Question

(a) Distinguish between kinetic energy and gravitational potential energy.

(b) A particle possesses energy in two forms only: kinetic energy and gravitational energy. It has a total energy of 3.0 J and is initially at rest. Its potential energy E_p changes causing a corresponding change in its kinetic energy E_k. No external work is done on or by the system. Copy the grid on the next page and draw a graph of kinetic energy E_k against potential energy E_p.



Explain how your graph illustrates the principle of conservation of energy.

(c) An AS Physics student plans to enter for the high jump event at the School Sports.

She estimates that, if she is to have a chance of winning, she will have to raise her centre of mass by 1.6 m to clear the bar. She will also have to move her centre of mass horizontally at a speed of 0.80 ms^{-1} at the top of her jump in order to roll over the bar.

(i) The student's mass is 75 kg. Estimate the total energy required to raise her centre of mass and roll over the bar.

(ii) The student assumes that this energy can be supplied entirely from the kinetic energy she will have at the end of her run-up. Estimate the minimum speed she will require at the end of the run-up.

[CCEA AS Physics 2006]

Power

Power is defined as the rate of doing work.

The definition can be expressed as an equation:

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken}}$$

or

$$P = \frac{W}{t}$$

where P = power in watts (W) or joules per second (Js^{-1})
W = work done in joules (J)
t = time taken in seconds (s)

If the work is being done by a **constant** force, F, then we know that the work done, W, can be written as W = Fs. Making this substitution for W in the power equation gives P = Fs ÷ t. If the **displacement occurs at a steady rate**, then speed, v = s ÷ t and we arrive at:

P = Fv

where P = power in watts (W) or joules per second (Js^{-1})
F = force being applied in newtons (N)
v = constant speed at which force is moving in ms^{-1}

Worked Examples

1 An electric motor has an output power of 2400 W and is used to raise a ship's anchor. If the tension in the cable is 8 kN, at what constant speed is the anchor being raised?

Solution

P = Fv, so v = P ÷ F = 2400 ÷ 8000 = **0.3 ms^{-1}**

2 A car of mass 1200 kg has an output power of 60 kW when travelling at a speed of 30 ms^{-1} along a flat road. What power output is required if the same car is to travel at the same speed up a hill of gradient 10%? (Such a hill has an angle of slope of $\tan^{-1}(0.1)$ or 5.7°.)

Students are advised to re-visit the material on resolution of forces on the inclined plane (page 35) before attempting this question.

Take g as 9.81 ms^{-2}.

Solution

Additional force to be overcome due to hill = mg.sin θ = 1200 × g × sin 5.7° = 1169.19 N
Additional power required = Fv = 1169.19 × 30 = 35 076 W ≈ 35 kW

Total power required = 60 + 35 = 95 kW

3 The engine of a motor boat delivers 36 kW to the propeller while the boat is moving at 9 ms^{-1}. Calculate the tension in the tow rope if the boat were being towed at the same speed.

Solution

Force (tension) = P ÷ v = 36000 ÷ 9 = 4000 N = 4 kN

4 The dam at a certain hydroelectric power station is 170 m deep. The electrical power output from the generators at the base of the dam is 2000 MW. Given that 1 m^3 water has a mass of 1000 kg, calculate the minimum rate at which water leaves the dam in m^3s^{-1} when electrical generation takes place at this rate.

Why is this figure the **minimum** rate of flow? Take g as 9.81 ms^{-2}.

Solution

In 1 s, the potential energy converted to electrical energy is 2×10^3 MJ = 2×10^9 J

Gravitational p.e. = mgh = m × 9.81 × 170 = 1667.7 × m

So mass removed from dam **every second** = $(2 \times 10^9) \div 1667.7 \approx 1.20 \times 10^6$ kg

So rate of flow = $(1.2 \times 10^6) \div 1000 = 1200$ m^3s^{-1}

Calculated flow rate is a **minimum** because it has been assumed that all the gravitational potential energy has been converted into electrical energy and no allowance has been made for the wasted heat and sound energy.

Efficiency

Efficiency is a way of describing how good a device is at transferring energy from one form to another in an intended way.

If a light bulb is rated 100 W, this means that it normally uses 100 J of electrical energy every second. But it might only produce 5 J of light energy every second. The other 95 J are wasted as heat. This means that only 5% of the energy is transferred from electrical energy into light energy. This light bulb therefore has an efficiency of 0.05 or 5%. If the same light bulb were used as a heater, its efficiency would be 95% or 0.95, because the intended output energy form would be heat, not light.

Opposite are two equivalent equations which can be used to define efficiency. Since efficiency is a ratio of two quantities each with the same unit, efficiency itself is dimensionless, that is, **efficiency has no unit.**

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}}$$

or

$$\text{efficiency} = \frac{\text{useful energy output in a given time}}{\text{total energy input in the same time}}$$

Worked Examples

1 A filament lamp rated 60 W has an efficiency of 0.04 (4%). A modern long-life lamp is rated 12 W and produces the same useful output power as the filament lamp. Calculate (a) the useful output power of the filament lamp and (b) the efficiency of the long-life lamp.

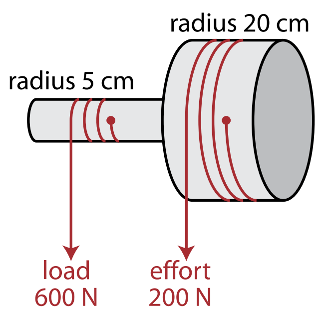
Solution

(a) useful output power = efficiency × total input power = 0.04 × 60 = 2.4 W

(b) $\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}} = \frac{2.4}{12} = 0.2 = 20\%$

2 A wheel-and-axle is a simple machine in which a small effort force can be used to raise a heavy load. In the wheel-and-axle shown in the diagram, a rope under 200 N tension (the effort) is wrapped around a ‘wheel’ of radius 20 cm to raise a load of weight 600 N.

Calculate the machine’s efficiency.



Solution

When the wheel rotates once, the effort falls a distance of $(2 \times \pi \times 0.2)$ metres and at the same time the load rises through a distance of $(2 \times \pi \times 0.05)$ metres.

For every revolution, the work done by the effort = Fs = $200 \times (2 \times \pi \times 0.2) = 80\pi$ joules and the work done on the load = Fs = $600 \times (2 \times \pi \times 0.05) = 60\pi$ joules

$$\text{efficiency} = \frac{\text{useful energy output in a given time}}{\text{total energy input in the same time}} = \frac{60\pi}{80\pi} = 0.75 = 75\%$$

Exercise 15

Where relevant take g as 9.81 ms^{-2} and give your answer to an appropriate number of significant figures.

1 To enable a train to travel at a steady speed of 30 ms^{-1} along a level track, the engine must supply a pulling force of 50 kN.

(a) How much work is the engine doing every second?

(b) If the power is proportional to the cube of the velocity, how much power is needed to drive the train at a speed of 40 ms^{-1}?

2 A car of mass 600 kg moves at a constant speed of 20.0 ms^{-1} up an inclined road which rises 1 m for every 40.0 m travelled along the road. Calculate

(a) the constant kinetic energy of the car and

(b) the rate at which the gravitational potential energy of the car is increasing.

3 A lorry of mass 35 000 kg moves at a constant maximum speed, v, up an inclined road which rises 1.00 m for every 10.0 m travelled along the road. The output power of the engine is 175 kW. Calculate (a) the value of v, if friction forces can be ignored and
(b) the value of v, if the friction force is 4665 N.

4 A simple pendulum has a length of 1.00 m. The bob is pulled to one side so that the angle between the taut string and the vertical is 60.0°. The pendulum is then released.

(a) Why can Newton's equations of motion not be applied in this situation?

(b) Show that the maximum speed of the pendulum in its motion is 3.13 ms^{-1}.

5 Several stones are projected upwards with the same initial speed, u, but at different angles α ($\alpha > 0$) to the horizontal. A student claims that at any common height reached by all of the stones, the *speed* of each stone is the same. Is the student right?

A stone projected at 15.0 ms^{-1} at an unknown angle α ($\alpha > 0$) to the horizontal.

Show that when it is 2.00 m above the ground the stone's speed is 13.6 ms^{-1}

Exercise 16

1 You lift your schoolbag from the floor to your desk, and leave it resting there. Describe the energy changes that take place. Explain also how the principle of conservation of energy applies to this operation.

[CCEA AS Physics January 2002]

2 A model helicopter of mass 0.60 kg, initially at rest on the ground, rises vertically into the air with uniform acceleration. At a height of 35 m above the ground its speed is 5.9 ms^{-1}.

(a) calculate the change in kinetic energy of the helicopter as it rises from the ground to a height of 35 m.

(b) calculate the change in gravitational potential energy of the helicopter as it rises to this height.

[CCEA AS Physics June 2003]

3 (a) (i) State the principle of conservation of energy.

(ii) A rugby ball is kicked from the ground towards the goalposts directly against the wind. The ball rises from ground level to a maximum height, and then falls. Explain how the principle of conservation of energy applies to this situation from the instant that the ball leaves the ground until it reaches its maximum height. Air resistance cannot be neglected.

(b) In this part of the question, neglect air resistance. An object falls from rest from a certain height H to the ground. As it falls, its potential energy E_p and its kinetic energy E_k both change.

(i) Sketch a graph to show the potential energy E_p varies with time from the moment of release until it reaches the ground.

(ii) Sketch a graph to show the kinetic energy E_k varies with time from the moment of release until it reaches the ground.

(iii) Sketch a graph to show the kinetic energy E_k depends on the height h of the object above the ground until it reaches the ground.

[CCEA June 2002]